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CALCULATING THE SPEED WITH WHICH WATER-PERMEABLE GROUND IS FROZ--ETC(U)
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CALCULATING THE SPEED WITH WHICH WATER-PERMEABLE GROUND IS FROZEN BY A ROW OF COLUMNS AFTER FROZEN GROUND CYLINDERS JOIN

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A.I. Pekhovich

CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
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This article examines the problem of ground freezing after the frozen ground cylinders join. In examining this problem only one area has a solution, when freezing columns are located in a single straight row and the filtering flow after the frozen ground cylinders join is directed along the frozen ground wall.

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NOTICE

1. Introduction

In the Soviet Union the method of artificially freezing ground is being used more and more every year in the construction industry. Under complex hydrogeological conditions this method makes it possible to create water-impermeable retaining walls, cofferdams and arches, and ground can be reinforced. The area in which ground freezing can be applied includes all the basic branches of the construction industry. Some idea of the work which has been carried out can be gained from the fact that in the last 20 years in our country more than 1.5 million cubic meters of ground have been frozen [1].

It is necessary, however, to admit the fact that, despite the tremendous volume of the work which has been carried out, the level of our knowledge of freezing technology has not yet reached the necessary level. This fact became especially evident when the freezing method was used on several hydrotechnical construction projects where water-permeable soils were subjected to freezing.

Recent studies carried out by the Ice Thermics Laboratory of the B. Ye. Vedeneyev All-Union Scientific Research Institute of Hydraulic Engineering have shown that the methods used by design organizations to make thermal calculations of ground freezing and primarily of the freezing of water-permeable soils contained significant flaws. In this process it was found that one of the basic shortcomings is the extremely approximate allowance which is made for the thermal influx from the unfrozen ground to the surface of the frozen mass. When we examine the problem of freezing ground in which there is no water motion, this drawback means that the actual freezing time will not coincide with the projected data; however, freezing under these conditions is usually successful due to the fact that the intensity of the heat flow from the outside decreases with time. Therefore the existing methods of calculating the freezing rate of nonwater-permeable soils are adequate for engineering practice to a certain extent.

However, the picture is different when we discuss the freezing of water-permeable ground. In this case the thermal flux from the outside does not decrease with time, and as a result of this it may be found that a design solution based on calculation results in which the thermal flux from the outside is considerably lower than it is in reality will be proven wrong in practice (for instance, it will be impossible to make the frozen ground cylinders join or the assigned thickness of the frozen ground walls will not be achieved). Therefore it is natural to direct special attention at improving the methods of making thermal calculations of the freezing of water-permeable soils.

In 1951 the pages of the "B. Ye. Vedeneyev All-Union Scientific Research Institute of Hydraulic Engineering" published a work by B. V. Proskuryakov which provided a new method for thermally calculating the freezing of the water-permeable ~~ground~~ by a single column [2].

A subsequent experimental check made by the Institute of this calculation method proved that the author's equations were correct and simultaneously demonstrated the great imprecision of the formulas which had been used up to that time.

At the present time it is generally accepted that the speed with which ground is frozen by a group of columns before the frozen ground cylinders join is carried out without allowance for the mutual influence (interference) of the frozen ground cylinders, i.e., the freezing of the ground by one group column is generally equated with the freezing of ground by a single column. It must be noted that this is completely wrong. Thus, for instance, in reality when freezing nonwater-permeable ~~ground~~ a single column will have less favorable conditions than a group column in view of the fact that the frozen ground cylinder of a group column is partially screened with regard to heat by its neighboring cylinders and therefore in this case the thermal influx from the unfrozen ground will be less than in the case of a single frozen ground cylinder. In water-permeable ground the presence of neighboring cylinders also exerts an influence. We consider it very desirable that in the near future the problem of the mutual influence of freezing columns both in water-permeable and in nonwater-permeable soils be studied in detail. In the meantime, however, we will assume that the calculations of ground freezing rates before the solidification of the frozen ground cylinders will be carried without allowance for the influence of the above-mentioned column interference factor.

In this article we shall examine the problem of ground freezing after the frozen ground cylinders join. In doing so we will state and solve only a single partial problem: when the freezing columns are located in a single straight row and the filtering flow after the frozen ground cylinders join is directed along the frozen ground wall.

2. Statement of the Problem

The above-mentioned problem of creating a frozen ground wall can be formulated as follows.

A row of freezing columns is implanted in an infinite, homogeneous layer of water-permeable ground perpendicular to the layer (Figure 1). The top and bottom of the layer are two mutually parallel surfaces which are impermeable to water and heat. The length of each column is equal to the thickness of the layer h. Columns with radius r_0 are arranged with interval F between them. The temperature of the filtration flow at a considerable distance from the column is equal to T. After the frozen ground cylinders join, the filtration flow is directed along the frozen ground wall. The speed of the filtration flow is equal to v. The ground freezes at temperature t_0. The physical characteristics of the frozen and unfrozen ground are known. It is necessary to find a dependency which determines the freezing speed of the ground after the frozen ground cylinders join.

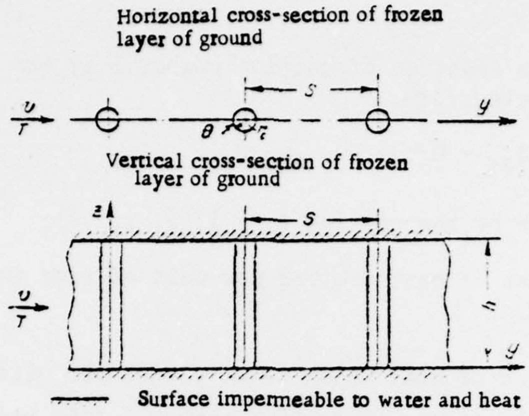

Figure 1. Diagram of Columns Arranged in a Row Parallel to the Filtration Flow.

In solving the problem, we mark out the path which we will use: we will first formulate in general form the heat balance equation of the frozen ground wall, we then determine the expressions of its components and substitute them into the initial equation; by integrating the differential equation which is obtained in this manner, we find the desired solution to the problem.

3. Heat Balance Equation and Its Components

a) Heat balance equation. The intensity of the heat influx from the unfrozen ground varies along the wall. Therefore the thickness of the *wall along its length is different* (Figure 2). Allowing for the fact that the thickness of the wall changes smoothly, it is possible to mentally divide the wall into sections of finite length and to assume that within the limit of each section it has a uniform thickness. If we select the number of sections to be equal to the number of columns, then the length of each section will be equal to S.

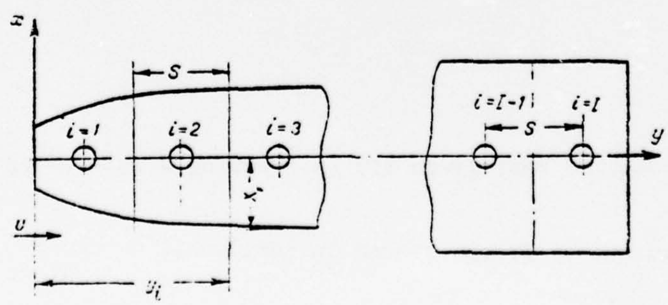

Figure 2. Water-Permeable Ground Frozen by a Row of Columns After the Cylinders Solidify.

The heat balance equation of section number i of the frozen ground has the following form:

$$Q_{fi} = Q_{ci} - Q_{fi\ i} - Q_{gi}, \tag{1}$$

where i is the number of the section (i = 1, 2, ... I);

Q_{fi} is the amount of heat evolved per unit of time in freezing water in section number i;

Q_{ci} is the amount of heat which is extracted per unit of time from the ground by the column in section number i (the heat absorbed by the freezing column);

$Q_{fi\ i}$ is the amount of heat which the filtration flow gives up per unit of time to the surface of the frozen ground cylinder in section number i;

Q_{gi} is the amount of heat which is extracted per unit of time from the frozen ground in section number i (the cooling of the frozen ground wall).

Thermal flow Q_{gi}, as will be shown later, can be taken into account by simply increasing the calculated value of the latent ice formation heat which belongs to the expression which determines the thermal flux Q_{fi}. Because of this fact heat balance equation (1) acquires the following similar form:

$$Q_{fi} = Q_{ci} - Q_{fi\ i}. \tag{2}$$

b) Heat given off at the freezing boundary. The amount of heat which is given off per unit of time in freezing the water in section i is equal to

$$Q_{fi} = \sigma \frac{dV_i}{d\tau}, \tag{3}$$

where

σ is the amount of heat given off in freezing a unit of ground volume;

V_i is the volume of frozen ground in section i;

τ is time.

Since

$$V_i = 2X_i hS - \pi r_0^2 h,$$

where $2X_i$ is the thickness of the frozen ground wall in section i, we have

$$dV_i = 2hSdX_i. \tag{4}$$

By substituting (4) into (3), we finally find

$$Q_{fi} = 2hS\tau \frac{dX_i}{d\tau}. \tag{5}$$

c) The heat influx from the unfrozen ground. The heat influx from the unfrozen ground in section i is equal to:

$$Q_{fi\,i} = 2h \int_{y_i}^{y_i+S} q_{fi}\, dy, \tag{6}$$

where

y_i is the distance from the beginning of the wall to the beginning of the area in question;

q_{fi} is the intensity of the thermal influx from the unfrozen ground to the wall at a distance of y from the beginning of the wall.

The intensity of the thermal influx Q_{fi} can be found in the following fashion.

We will write the equation of the established heat balance of the elementary volume of unfrozen ground, and we will then convert this equation into the following form:

$$\frac{\partial t}{\partial y} = \frac{\lambda_2}{c_w \gamma_w v}\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2}\right); \tag{7}$$

here

t is the temperature of the ground;

λ_2 is the thermal conductivity factor of the unfrozen ground;

c_w is the thermal capacity of the water;

γ_w is the specific weight of the water;

v is the speed of the filtration flow.

Assuming that the magnitude of the thermal flow which arises due to physical heat conductivity and is directed parallel to the wall (along axis y) is considerably less than the value of the heat flow which is transferred in the same direction by the water, it is possible to assume:

$$\frac{\partial^2 t}{\partial y^2} \approx 0;$$

then instead of (7) we will have

$$\frac{\partial t}{\partial y} = b_{fi} \frac{\partial^2 t}{\partial x^2}. \tag{8}$$

where

$$b_{fi} = \frac{\lambda_2}{c_w \gamma_w}.$$

We will ignore the curvature of the frozen ground wall surface and we will transfer the origin of the coordinate system to it; then

$$\begin{aligned} &\text{when } x = 0 \text{ we have } t = t_0, \\ &\text{when } x = \infty \text{ we have } t = T, \\ &\text{when } y = 0 \text{ we have } t = T. \end{aligned} \tag{9}$$

Equation (8) may be regarded as a one-dimensional thermal conductivity equation in which the time variable τ is replaced by the variable coordinate y. The solution to this equation under boundary conditions (9) is known in heat transmission theory and looks as follows [3, p. 71; 4, p. 91]:

$$t = (T - t_0)\, G\left(\frac{x}{\sqrt{4 b_{fi} y}}\right) + t_0, \tag{10}$$

where G is the integral of error probability.

From the last expression it follows that the temperature gradient in the direction of axis x is equal to

$$\frac{\partial t}{\partial x} = \frac{T - t_0}{\sqrt{\pi b_{fi} y}}\, e^{-\frac{x^2}{4 b_{fi} y}}.$$

At the surface of the frozen ground wall the following is true

$$\left.\frac{\partial t}{\partial x}\right|_{x=0} = \frac{T - t_0}{\sqrt{\pi b_{fi} y}}.$$

Consequently, the intensity of the thermal influx from the unfrozen ground to the wall will be

$$q_{fi} = \frac{\lambda_2 (T - t_0)}{\sqrt{\pi b_{fi} y}}. \tag{11}$$

Equation (11) was formulated in 1945 by B. V. Proskuryakov and was used by him in solving the problem of freezing water-permeable ground [5].

By substituting (11) into (6) and by performing integration, we finally find the desired equation for determining the magnitude of the thermal influx from the unfrozen ground to the wall in section number i:

$$Q_{fi\,i} = \frac{4h\lambda_2 (T - t_0)}{\sqrt{\pi b_{fi}}} \left(\sqrt{y_i + S} - \sqrt{y_i}\right). \tag{12}$$

From equation (12) it follows that the thermal influx from the unfrozen ground does not change during freezing, i.e., the heat influx is a constant value for every section.

d) <u>The absorption of heat by freezing columns</u>. The heat absorbed by each of the freezing columns in the row depends on the temperature of the brine circulating in the column and on the thermal resistance of frozen ground in the area in question. Since the speed with which the freezing boundary moves is extremely slow, the temperature distribution in the frozen ground is quasistationary. Therefore, in the general case we have

$$Q_{ci} = \frac{t_0 - \theta}{\Psi_i}, \tag{13}$$

where Ψ_i is the thermal resistance of the frozen ground between the column and the freezing boundary in section number i.

By utilizing the known solution to the hydromechanical problem of fluid flow to an infinite row of ideal holes arranged in a band-like stratum [6, p. 35], we can write

$$\Psi_i = \frac{n + X_i}{2\lambda_1 hS}, \tag{14}$$

where n is the half-thickness of an imaginary wall which corresponds to the additional thermal resistance which arises due to the fact that temperature θ is not maintained over the entire surface in which the row of columns is located, but only on the surface of the columns which are some distance apart:

$$n = \frac{S}{\pi} \ln \frac{S}{2\pi r_0}; \tag{15}$$

λ_1 is the thermal conductivity factor of the frozen ground.

Value n is fairly considerable. Thus at column radius r_0 = 0.05 m and when the distance between the columns is S = 1 m, according to (15) we find that n = 0.27 m. Therefore we cannot ignore the value n.

In order for equation (13) which expresses the heat absorption of the column to be single-valued and absolute, it is necessary to again examine the value of brine temperature θ.

As is well known, the temperature of the brine changes during freezing. These changes are completely regular and are a function of the characteristics of the refrigeration device and the increase during freezing in the thermal resistance of the frozen ground. Without making a detailed examination here of the problem of calculating the course of the brine temperature, we will simply point out certain general facts in this area.

The operative refrigeration capacity of compression refrigeration devices primarily depends on the evaporation temperature of the coolant

and consequently on the temperature of the brine. On the other hand, the thermal absorption of the columns also depends on the brine temperature. The operative refrigeration capacity of the devices should always be equal to the thermal absorption of the column. This equality occurs (or more precisely speaking, arises) at a certain brine temperature. As the volume of the frozen ground mass increases and consequently as the thermal resistance of the frozen ground increases, this equilibrium is disrupted since the increased thermal resistance causes a reduction in the columns' heat absorption. The production of cold begins to exceed the requirement; the difference decreases the temperature of the brine. Equilibrium is restored at a different lower brine temperature. This is how we find the dependence which is correct when the columns are arranged in any form:

$$\theta = f(\Psi). \tag{16}$$

When the columns are arranged in a row, after the frozen ground cylinders join dependency (16) acquires the following form:

$$\theta = f(X). \tag{17}$$

The reduction in brine temperature has a limit which is determined by the technical operating conditions of the refrigeration devices. When this limit, which corresponds in each case to a certain frozen ground thermal resistance value, is reached, subsequent freezing will take place at a constant minimum brine temperature:

$$\theta = \theta_{min}. \tag{18}$$

From what we have stated it follows that the freezing process is broken down into two periods. During the first, initial period the temperature of the brine decreases, and during the second period the brine temperature is constant.

The transition from the first period to the second is determined by substituting (18) into (17):

$$\theta_{min} = f(X_{I-II}),$$

where $2X_{I-II}$ is the value of the wall thickness at which the first period of freezing stops and the second period of freezing begins.

It must be noted that this transition to the second period of freezing occurs in the majority of cases even before the cylinders join, and therefore freezing after joining usually occurs at a constant brine temperature.

e) Consideration of the quantity of heat extracted from the frozen ground wall when it is cooled. We will determine the quantity of heat which is extracted per unit of time from the frozen ground in section number i when the wall is cooled. In doing so we will assume that temperatures θ is not maintained on the column surface, but rather

on the plane in which the row of columns is arranged. In examining expressions (13) and (14) which determine the amount of heat absorbed by the columns, it was pointed out that this assumption should not be adopted. The basis for introducing this assumption in this case which considerably simplifies the mathematical computations, is the following considerations: 1) the value of component Q_{gi} is usually small in comparison with the other components of heat balance equation (1); 2) this assumption obviously leads only to a small increase in the calculated value of Q_{gi} and consequently to a very small increase in the calculated value of freezing time; the latter fact provides a certain margin in designing (the time calculated for the formation of a frozen ground wall increases).

Thermal flux Q_{gi} is equal to:

$$Q_{gi} = \frac{dW_i}{d\tau},$$

where W_i is the heat content of section i of the frozen ground wall.

Since

$$W_i = 2hc_1\gamma_1 S \frac{t_0 - \theta}{2} X_i,$$

then it is obvious that

$$Q_{gi} = c_1\gamma_1 hS(t_0 - \theta) \frac{dX_i}{d\tau}, \tag{19}$$

where c_1 and γ_1 are the heat capacity and specific weight of the frozen ground.

Now let us compare expression (19) with equation (5) which determines the magnitude of thermal flux from the latent ice formation heat. It is easy to see that if we stipulate

$$\sigma' = \sigma + c_1\gamma_1 \frac{t_0 - \theta}{2}, \tag{20}$$

then

$$Q_{fi} + Q_{gi} = 2hS\sigma' \frac{dX_i}{d\tau} = \frac{\sigma'}{\sigma} Q_{fi}. \tag{21}$$

This is how we can allow for the effect of thermal flow Q_{gi} by increasing the calculated value of the latent ice formation heat σ.

4. Solving the Heat Balance Equation

For the first period of freezing where the brine temperature decreases, heat balance equation (1) acquires the following form when the expressions of its components (5) and (13) are substituted in it, with allowance for (17) and (21):

$$2hS\varepsilon' \frac{dX_i}{d\tau} = \frac{t_0 - f(X_i)}{\Psi_i} - Q_{fi}. \qquad (22)$$

The solution to the stated problem, determining the time required for the formation of a frozen ground wall, is found by integrating the last equation within the limits of $X_i = X_{i1}$ to $X_i = X_{i2}$ and $\tau = \tau_1$ to $\tau = \tau_2$:

$$\tau_2 - \tau_1 = 2hS\varepsilon' \int_{X_{i1}}^{X_{i2}} \frac{dX_i}{\frac{t_0 - f(X_i)}{\Psi_i} - Q_{fi}}. \qquad (23)$$

In the general case where function $f(X_i)$ which expresses the brine temperature is complex in form, it is simplest to calculate the value of the integral by the graphic or tabular method.

The maximum thickness of the frozen ground wall is determined by the equation which expresses the equality between heat absorption by the column and the thermal influx from unfrozen ground:

$$\frac{t_0 - f(X_i)}{\Psi_i} = Q_{fi}. \qquad (24)$$

In cases where it is possible to assume within certain limits that the column's heat absorption is constant, i.e., that

$$\frac{t_0 - f(X_i)}{\Psi_i} = Q_{ci} = const.$$

it is easy to find from (23) that:

$$X_{i2} = \frac{Q_{ci} - Q_{fi}}{2hS\varepsilon'} (\tau_2 - \tau_1) + X_{i1}. \qquad (25)$$

We will now derive a computation equation for the second freezing period where the brine temperature is constant. Heat balance equation (1) acquires the following form:

$$2hS\varepsilon' \frac{dX_i}{d\tau} = \frac{2\lambda_1 hS (t_0 - \theta_{min})}{X_i + n} - Q_{fi}, \qquad (26)$$

or, by separating the variables we find:

$$d\tau = \frac{X_i + n}{B(X - X_i)} dX_i,$$

where

$$B = \frac{Q_{fi}}{2hS\varepsilon'}; \qquad (27)$$

$$X = \frac{2hS\lambda_1 (t_0 - \theta_{min})}{Q_{fi}} - n. \qquad (28)$$

By integrating the latter differential equation within the limits of $X_i = X_{i1}$ to $X_i = X_{i2}$ and $\tau = \tau_1$ to $\tau = \tau_2$, we obtain the following calculation equation:

$$\tau_2-\tau_1=\frac{1}{B}\left[(N+n)\ln\frac{N-X_{i1}}{N-X_{i2}}-(X_{i2}-X_{i1})\right]. \tag{29}$$

From equation (29) it is evident that the maximum value of the half-thickness of the wall is equal to:

$$X_{i2\,\max}=N. \tag{30}$$

If the initial value is $X_{i1} > N$, then obviously formula (29) will indicate the melting time of the wall, and the maximum value of the half-width of the wall, as before, will be equal to N.

In order to make computations easier, in Figures 3 and 4 we show computation graphs plotted according to formula (29).

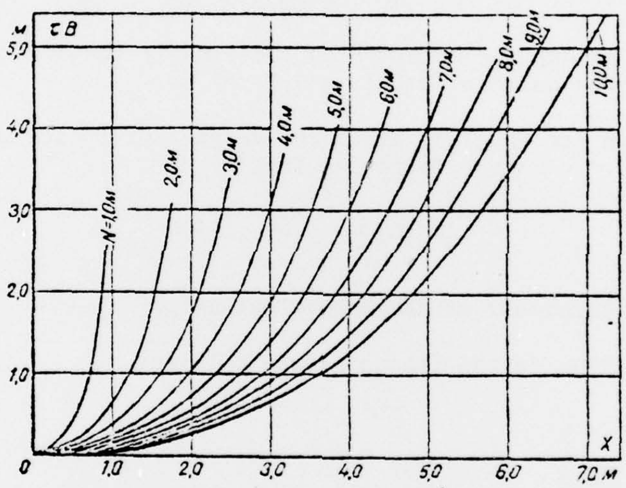

Figure 3. Graph for Determining the Time Required for the Formation of a Frozen Ground Wall (n = 0.37 m).

5. Replacing a Row of Closed Frozen Ground Cylinders with an Equivalent Wall

In the computation equations derived in the previous paragraph, the initial wall thickness was arbitrary. We are interested in the speed of freezing after the time when the frozen ground cylinders joined. Therefore we need to determine the wall thickness during this time. We will assume that instead of the actual wall which is composed of cylinders, we are dealing with a flat rectangular cross--section wall. We will determine what thickness of this "equivalent" wall should be. In doing so it is necessary to observe two conditions which consist of the fact that the volume of frozen ground and the thermal resistance between the column and the surface of the frozen ground must remain constant.

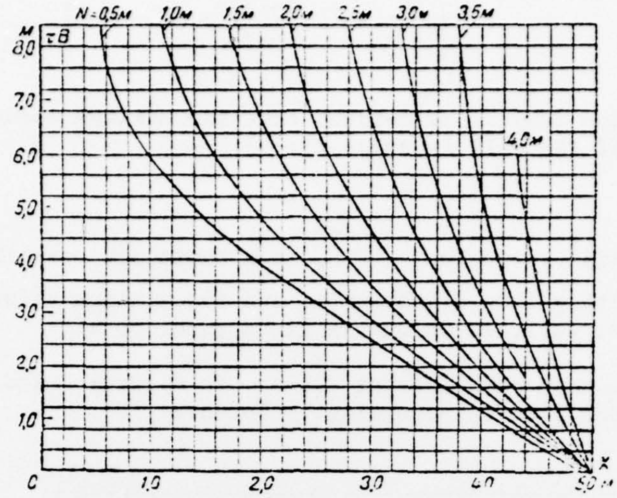

Figure 4. Graph for Determining the Time it Takes for a Frozen Ground Wall to Melt (n = 0.37 m).

The condition for equality among the volumes of frozen ground (Figure 5) is written in the following form:

$$(2X_0 S - \pi r_0^2)\, h = \pi (R^2 - r_0^2)\, h, \qquad (31)$$

where $2X_0$ is the thickness of the equivalent wall,

R is the radius of the frozen ground cylinder.

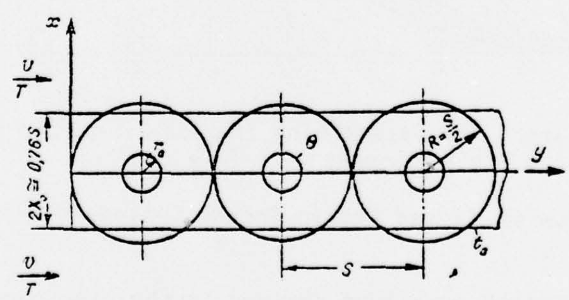

Figure 5. Ground Frozen by a Row of Columns at the Moment the Cylinders Join.

Keeping in mind the fact that at the moment of joining

$$R = \frac{S}{2},$$

we find from equation (31) that in accordance with the condition that the volumes of frozen ground are equal, the thickness of the equivalent wall is equal to

$$2X_0 = \frac{\pi}{4} S \approx 0{,}79S. \tag{32}$$

We will write an equation which corresponds to the second condition. The thermal resistance of the frozen ground cylinder Ψ_c is equal to

$$\Psi_c = \frac{1}{2\pi\lambda_1 h} \ln \frac{R}{r_0};$$

consequently, at the moment the cylinders join

$$\Psi_c = \frac{1}{2\pi\lambda_1 h} \ln \frac{S}{2r_0}. \tag{33}$$

The thermal resistance of the equivalent wall Ψ_r with a length which is equal to the distance between the column axes S can be presented in the following form in accordance with equation (14):

$$\Psi_r = \frac{X_0 + n}{2\lambda_1 h S}. \tag{34}$$

In accordance with the second condition $\Psi_c = \Psi_r$.

By substituting expressions (33) and (34) into this equation, we find

$$X_0 = \frac{S}{\pi} \ln \frac{S}{2r_0} - n$$

or, keeping in mind the fact that

$$n = \frac{S}{\pi} \ln \frac{S}{2\pi r_0},$$

we obtain

$$X_0 = \frac{S}{\pi}\left(\ln \frac{S}{2r_0} - \ln \frac{S}{2\pi r_0}\right);$$

from this we finally find that in accordance with the second condition the thickness of the equivalent wall may be equal to:

$$2X_0 = \frac{2\ln\pi}{\pi} S \approx 0{,}73S. \tag{35}$$

A comparison of expressions (32) and (35) shows that they provide results which diverge by less than 8%. Therefore, by satisfying the above-mentioned two conditions fairly precisely, we can assume:

$$X_0 = 0{,}38S. \tag{36}$$

From this expression it follows that the thickness of an "equivalent" wall is approximately equal to three-fourths of the distance between the axes of the freezing columns.

6. Sample Calculation

We will determine the time it takes for a frozen ground wall to form beginning at the time when the cylinders join and before the design thickness, equal to two meters, is reached.

The following are known:

speed of filtration flow.................. $v = 0.05$ m/hr
temperature of filtration flow............ $T = 4°C$
freezing temperature of ground............ $t_0 = 0°C$
specific weight of frozen ground.......... $\lambda_1 = 1600$ kg/m^3
thermal conductivity factor of frozen ground............................ λ_1 = kcal/m·hr·deg
heat capacity of frozen ground............ $c_1 = 0.35$ kcal/kg·deg
thermal conductivity factor of unfrozen ground.......................... $\lambda_2 = 1$ kcal/m·hr·deg
latent ice formation per unit of ground volume............................ $\sigma = 23{,}000$ kcal/m^3
heat capacity of water.................... $c_w = 1$ kcal/kg·deg
specific weight of water.................. $\lambda_w = 1$ kg/m^3
radius of freezing column................. $r_0 = 0.05$ m
length of one freezing column............. $h = 10$ m
distance between column axes.............. $S = 1$ m
number of freezing columns................ $I = 25$.

Freezing occurs during the second period when the brine temperature is constant $\theta_{min} = -20°C$.

Let us primarily examine the first section ($i = 1$).

According to (12) the heat influx from the unfrozen ground is equal to:

$$Q_{fi1} = \frac{4h\lambda_2(T - t_0)}{\sqrt{\pi k_{fi}\tau}}\left(\sqrt{y_i + S} - \sqrt{y_i}\right) = \frac{4\cdot 10\cdot 1\cdot 4\cdot\sqrt{1}}{\sqrt{\pi\frac{1}{1\cdot 1000\cdot 0.05}}} = 640\ \frac{\text{kcal}}{\text{hr}}.$$

According to expression (15) we find:

$$n = \frac{S}{\pi}\ln\frac{S}{2\pi r_0} = \frac{1}{\pi}\ln\frac{1}{2\pi\cdot 0.05} = 0.37\ m.$$

Now from (28) we can determine the value of N:

$$N = \frac{2hS\lambda_1(t_0 - \theta)}{Q_{fi1}} - n = \frac{2\cdot 10\cdot 1\cdot 2\cdot 20}{640} - 0.37 = 0.88\ m.$$

The maximum wall thickness is equal to $2X|_{\tau\to\infty} = 2N = 1.76\ m$, i.e., less than expected.

Let us examine the fifth section (i = 5). In the same way as we calculated the first section, we find

$$Q_{fl5} = \frac{4 \cdot 10 \cdot 1 \cdot 4}{\sqrt{\pi \frac{1}{1 \cdot 1000 \cdot 0{,}05}}} (\sqrt{4+1} - \sqrt{4}) = 154 \ \frac{\text{kcal}}{\text{hr}};$$

$$N = \frac{2 \cdot 10 \cdot 1 \cdot 2 \cdot 20}{154} - 0{,}37 = 4{,}83 \ м.$$

Here in the fifth section the maximum thickness

already considerably exceeds the expected value 2X = 2 m. From formula (29) we will determine the necessary time of active freezing. However, we first note that according to expression (36) the initial wall thickness is equal to $2X_0$ = 0.76 S = 0.76 m; the value of σ' is determined from equation (20)

$$\sigma' = \sigma + c_1 \gamma_1 \frac{t_0 - \theta}{2} = 23000 + 0{,}35 \cdot 1600 \ \frac{20}{2} = 28600 \ \text{kcal/m}^3.$$

According to expression (27) the value of B is equal to:

$$B = \frac{Q_{fl5}}{2hS\sigma'} = \frac{154}{2 \cdot 10 \cdot 1 \cdot 28600} = 2{,}69 \cdot 10^{-4} \ \text{m/hr}.$$

Keeping in mind the fact that $X_1 = X_0$ = 0.38 m and X_2 = 1 m, according to formula (29) we find:

$$\tau = \frac{1}{B}\left[(N+n) \ln \frac{N - X_1}{N - X_2} - (X_2 - X_1)\right] =$$

$$= \frac{1}{2{,}69 \cdot 10^{-4}}\left[(4{,}83 + 0{,}37) \ln \frac{4{,}83 - 0{,}38}{4{,}83 - 1} - (1 - 0{,}38)\right] = 595 \ \text{hrs} \approx 25 \ \text{days}.$$

Let us examine the last section of the wall (i = 25). We find:

$$Q_{fl25} = 640\left(\sqrt{24+1} - \sqrt{24}\right) = 64 \ \text{kcal/hr};$$

$$N = \frac{800}{64} - 0{,}37 = 12{,}13 \ м;$$

$$B = \frac{64}{572000} = 1{,}12 \cdot 10^{-4} \ \text{m/hr};$$

$$\tau = \frac{10000}{1{,}12}\left[(12{,}13 + 0{,}37) \ln \frac{12{,}13 - 0{,}38}{12{,}13 - 1} - (1 - 0{,}38)\right] = 495 \ \text{hrs} \approx 21 \ \text{days}.$$

This calculation shows that the difficulty consists of forming a frozen ground wall in the first (initial) section. It is obvious that here special measures should be undertaken, for instance, the implantation of several additional freezing columns. As a whole, however, the formation of a frozen ground wall after joining of the cylinders will take approximately 25 days.

join.

7. Conclusions

The main results of this article amount to the following.

1. Calculated dependencies are obtained which make it possible to determine the freezing and thawing time of a frozen ground wall which, after frozen ground cylinders are formed, is located parallel to the motion of the filtration flow:

a) for the first freezing period when the brine temperature changes, computation equation (23); in the special case where the heat absorption of the columns is constant, computation equation (25);

b) for the second freezing period where the brine temperature is constant, computation equation (29).

These computation equations differ from similar formulas mainly due to the fact that they make greater allowance for the values of thermal influx from the unfrozen ground and the heat absorbed by the columns.

2. Computation equations (24) and (30) determine the maximum value of the wall thickness towards which it will tend when freezing (or thawing) takes an infinite amount of time.

3. The intensity of the thermal influx from unfrozen water-permeable ground varies along the length of the wall; as water moves along the wall, the water is cooled and the intensity of the thermal influx decreases. From this it follows, in the first place, that it is most difficult to create the initial (with regard to the flow) section of the frozen ground wall, and in the second place that the creation of the wall must begin with the upper sections since this makes it easier to create the lower sections.

4. The value of quantity $A = \sqrt{\lambda c_w \gamma_w v}\,(T - t_0)$ may make it possible to make a preliminary evaluation of the difficulty of freezing water-permeable soils. The greater the value of A, the more difficult it is for freezing to take place. At values A = 100-120 kcal/m$^{3/2}$·hr and above, freezing is associated with considerable difficulties and therefore may prove to be economically unfeasible, as well as sometimes even technically impossible.

In conclusion I would like to note that in writing this article Professor (Doctor of Technical Science) B. V. Proskuryakov provided many valuable comments.

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